

Robust multipartite Bell tests with a single photon

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Lately, much interest has been directed towards designing setups that achieve decisive tests of local realism. Here we present a Bell test based on the state of a single nonlocalised photon. The scheme is easily extendible to any number of observers, allowing observation of multipartite nonlocality, and has a good tolerance to loss, compatible with current efficiencies of detectors and single-photon sources. The required entangled state and measurements can be realized via linear optics and single-photon detection, facilitating experimental implementation of the scheme. We also consider the case of atom-photon entanglement, where the loss threshold can be lowered further, as well as local filters compensating transmission and coupling inefficiencies at the source.

Introduction.—Experiments performed by space-like separated, independent observers may display correlations that do not comply with assumptions of local realism, i.e. that physical quantities have well established values prior to measurement and that signal propagation is restricted by the speed of light in accordance with special relativity. Such assumptions lead to restrictions, usually expressed as Bell inequalities [1], which can be surpassed within quantum theory. Correlations which violate a Bell inequality are referred to as nonlocal. From an applied point of view, nonlocality represents a physical resource, which enables protocols such as device-independent quantum key distribution [2] and random number generation [3]. These protocols rely on the violation of a Bell inequality to ensure secrecy and randomness, without further assumptions about their physical implementation.

In spite of steady theoretical and experimental progress, nonlocality has yet to be demonstrated in a loophole free manner. All experiments to date suffer from either the locality loophole, meaning that the assumption of space-like separation of the observers is not fulfilled [4], or the detection loophole, which is opened when the efficiency of the detectors employed in the Bell test is insufficient [5]. Both loopholes open for local-hidden-variable explanations of the observed data and compromise the security of cryptographic protocols. Closing the loopholes is important both from a fundamental and a technological perspective. To close the locality loophole, it is advantageous to work with optical systems since light can be distributed with relative ease among spatially separated parties and since optical detectors are fast. Various approaches have been considered towards closing the detection loophole in optical Bell tests. Two fundamental types of entangled states which may be used are polarisation-entangled states of fixed photon number, or states relying on superposition of one or few photons with the vacuum. Both approaches are hampered by the low efficiency of most available single-photon detectors [6] (although high-efficiency superconducting transition-edge sensors are slowly becoming more widespread). The former case has the advantage that projective measure-

ments in any basis can be performed with linear optics. However, heralded production of entangled photon pairs is so far only possible with low efficiency [7], incompatible with the critical thresholds required for a bipartite Bell test. Theoretically, it has been shown that increasing the number of parties N can improve the thresholds [8, 9]. E.g. multipartite Bell tests based on GHZ states tolerate a detection efficiency approaching 50% for large N [9]. However, producing GHZ states of more parties also becomes increasingly demanding [10] and larger GHZ states are more sensitive to noise [11]. For the case of photon-number superpositions, entanglement generation can be achieved with relatively large efficiency since, in the simplest case, it suffices to split a single photon on a beam splitter. The disadvantage is that perfect projective measurements are not available in all bases, using linear optics and photon counting.

Here we demonstrate that using the simple, displacement-based measurements of Ref. [12], it is possible to attain good efficiency thresholds which almost coincide with the thresholds for perfect projections. Our scheme is based on the state of a single photon split between multiple modes, i.e. a single-photon W-state [13] which is known to be extremely robust against losses [14]. The state is simple to produce for any number of parties, and hence the scheme is well suited for testing multipartite nonlocality. For an all-optical state, we show that a combined efficiency threshold for coupling, transmission, and detection of $\sim 68\%$ can be achieved for moderate to large N . We also show that for atom-photon entanglement with a single atomic party, the threshold can be as low as 43.7%. Furthermore, we suggest a method to compensate transmission and coupling losses at the source by local filtering. Surprisingly, the use of filters allows Bell inequality violations for arbitrarily low atom-photon coupling efficiency.

W-States and full correlator inequalities.—Throughout most of the paper, we will consider a scenario with N parties, each party k having a choice of two dichotomic measurements $A_0^{(k)}$, $A_1^{(k)}$, as illustrated in Fig. 1(a). For this scenario, there are 2^{2^N} tight, linear, full-correlator Bell

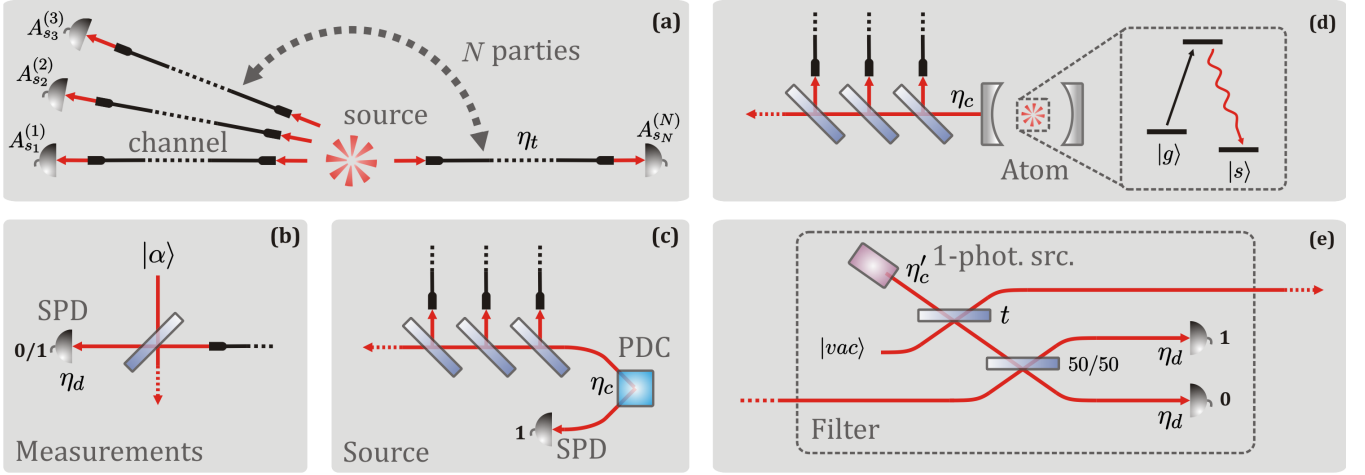


FIG. 1. (Colour online) (a) A source distributes a single photon to N parties through lossy channels of transmissivity η_t . Each party chooses between two measurements with binary outcomes. (b) Measurements consists in a variable displacement, followed by single-photon detection by bucket detectors of efficiency η_d . The displacement controls the measurement setting. (c) W-state creation based on a possible single-photon source using parametric down conversion. Detection of light in one output arm heralds the presence of a photon in the other arm with a coupling efficiency η_c . The single photon is divided among N modes by beam splitters of suitable transmission. (d) Atom-photon entanglement. We consider an atom with a λ -type level scheme. The atom is initially prepared in one of two stable ground states $|g\rangle$. An entangled state between atom and light is then generated by partly exciting the atom allowing decay into $|s\rangle$ accompanied by emission of a photon. The photon is coupled out of the cavity and distributed to the remaining $N - 1$ parties. (e) Single-photon amplifier as a filter. Two auxiliary modes are used, one of which must contain a single photon. Success is conditioned on a click in one detector only. When the transmissivity t is small, the single-photon component of the output is amplified, compensating preceding losses.

inequalities, all of which can be compactly expressed in a single, nonlinear inequality, the Werner-Wolf-Weinfurter-Zukowski-Brukner (W^3ZB) inequality [15]

$$\sum_r |\tilde{\xi}(\mathbf{r})| \leq 1, \quad (1)$$

where $\tilde{\xi}(\mathbf{r}) = 2^{-N} \sum_{\mathbf{s}} (-1)^{\mathbf{r} \cdot \mathbf{s}} \xi(\mathbf{s})$, with \mathbf{r} and \mathbf{s} being vectors in $\{0, 1\}^N$ and $\xi(\mathbf{s}) = \xi(\mathbf{s}_1 \cdots \mathbf{s}_N) = \langle A_{s_1}^{(1)} \cdots A_{s_N}^{(N)} \rangle$ the corresponding full correlator. We will focus on the violation of (1) by W-states, i.e. states of the form

$$|W_N\rangle = \frac{1}{\sqrt{N}} (|1, 0, \dots, 0\rangle + \dots + |0, 0, \dots, 1\rangle). \quad (2)$$

Such states can be easily produced in the case where $|0\rangle$, $|1\rangle$ are optical Fock states, by splitting a single photon equally among N modes. When undergoing loss in each mode, this single-photon state is transformed into $\rho_N(\eta) = \eta|W_N\rangle\langle W_N| + (1 - \eta)|vac\rangle\langle vac|$, with $1 - \eta$ the probability of loosing the photon. In the case of qubits, ρ_N represents a W-state subject to amplitude damping and was studied in Ref. [8]. For perfect Pauli measurements, $\rho_N(\eta)$ violates (1) for any number of parties, and the critical η required for violation improves with N as $\tilde{\eta} = 2/(3 - N^{-1})$. A plot of this function is shown in Fig. 2. For large N , we have $\tilde{\eta} \rightarrow 66.7\%$. The measurements attaining $\tilde{\eta}$ are in the x-z plane of the Bloch sphere, i.e. they are projections onto states of the form $\cos(\theta)|0\rangle + \sin(\theta)|1\rangle$. For qubits encoded into

zero- and single-photon Fock states, such projections are non-trivial to perform unless θ is a whole multiple of $\pi/2$. They cannot be implemented exactly with passive linear optics and single-photon detectors, since linear optics cannot change the energy of a state. However, as we show in the following, good tolerance to losses can be obtained, even with imperfect projections, achieved by simple linear optics measurements.

Implementation and thresholds.—Our setup is illustrated in Fig. 1(a)-(c). A central source produces a single photon, which is split among N modes and distributed to the parties through lossy channels, e.g. optical fibres, of transmissivity η_t . Taking into account the initial coupling loss η_c , the state shared by the parties becomes $\rho_N(\eta_c\eta_t)$. The measurements employed by the parties are of the type suggested in Ref. [12], namely displacements followed by single-photon detection. Physically, the displacements are implemented by mixing the signal with a coherent state from a local oscillator on a beam splitter with very high transmission, as shown in Fig. 1(b). We take the single-photon detectors to have an efficiency η_d , and since there is always a most one photon at the input, they do not need to be number resolving. Different measurement settings correspond to different choices for the displacement. A lossy detector can be modeled as a perfect detector preceded by loss acting on the input signal. This loss can be commuted through the displacement, since any loss from the local oscillator can be compen-

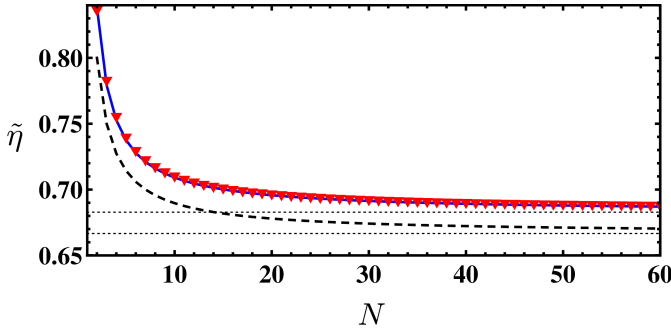


FIG. 2. (Colour online) Critical efficiency for increasing number of parties, shown for perfect Pauli measurements (dashed black) and displacement measurements (red triangles). The fit (solid blue) is indicated, as well as the asymptotic values of 0.667 and 0.683 (dotted).

sated by increasing the amplitude and is thus irrelevant. Hence, since the detector loss is the same for all parties and all settings, it is equivalent to transmission loss, and we can take it into account by modifying the state to $\rho_N(\eta_c\eta_t\eta_d)$. It is a nice feature of our scheme that all the losses are equivalent, allowing description by a single efficiency parameter $\eta = \eta_c\eta_t\eta_d$.

For a measurement by a perfect single-photon detector preceded by a displacement $D(\alpha)$, the no-click outcome corresponds to the projector $P_0 = D(\alpha)|0\rangle\langle 0|D^\dagger(\alpha) = |\alpha\rangle\langle\alpha|$, where $|\alpha\rangle$ denotes a coherent state. Assigning $+/-1$ to the no-click and click events respectively, the measurement operator is then given by $M(\alpha) = P_0 - (1 - P_0) = 2|\alpha\rangle\langle\alpha| - 1$, with matrix elements in the Fock basis

$$M_{mn}(\alpha) = 2e^{-|\alpha|^2} \frac{\alpha^n (\alpha^*)^m}{\sqrt{n!m!}} - \delta_{nm}. \quad (3)$$

Using this expression, for the simplest case where all parties use the same settings $\{\alpha_0, \alpha_1\}$, we analytically compute the full correlator $\xi(\mathbf{s})$ in the state $\rho_N(\eta)$. We then numerically maximize over $\{\alpha_0, \alpha_1\}$ to obtain the critical efficiency $\tilde{\eta}$ below which no violation of W^3ZB can be obtained. Due to the permutation symmetry of $\rho_N(\eta)$, the exponentially many terms in the sum (1) reduce to a number polynomial in N , enabling evaluation for relatively large N . We have verified for $N \leq 10$, that allowing different settings for each party does not improve the critical efficiency $\tilde{\eta}$.

The result of our maximisation is shown in Fig. 2. The qualitative behaviour of $\tilde{\eta}$ is similar to the ideal case of perfect Pauli measurements. For small N it decreases rapidly, then approaches an asymptotic value. Unfortunately, we have not been able to obtain an analytical expression for the limit, however, we have fitted a function of the form $a/(b - N^{-1})$ to our curve, and we find that the critical efficiency is well described by this when $a = 1.841$ and $b = 2.696$, which leads to an asymptotic value of $\tilde{\eta} \rightarrow 68.3\%$. The rapid decrease for small N is nice from

an experimental perspective, since it means that adding just a few beam splitters and detection stations, the loss tolerance for the Bell test can be significantly improved. For two parties, $\tilde{\eta} = 83.5\%$, which is close to the critical value 82.8% for testing the CHSH inequality with photon pairs in a maximally entangled polarisation state [16] (the present scheme still compares favourably, since the heralded production of ρ_N is significantly easier to achieve experimentally than heralded production of polarisation-entangled pairs [7, 17]). However, for $N = 10$ we already have $\tilde{\eta} = 70.9\%$. This represents a significant improvement over the schemes studied in Refs. [18, 19]. In particular, in [18] an additional use of a nonlinear crystal is required and due to the asymmetry between the measurements, transmission and detector efficiencies are not equivalent. Assuming perfect homodyne detection, and perfect coupling and transmission, the critical single-photon detector efficiency is 71.1%, comparable to our value. However, for a finite coupling and transmission efficiency $\eta_c\eta_t = 90\%$, the critical detector efficiency becomes 87% while in the present scheme it is 78.8%.

Atom-photon entanglement and filtering.—We now consider two possible variants of our scheme, which improve the tolerance to losses. For the first variant, we replace one of the parties by an atom in a cavity on which projective measurements in any basis can be performed with very high accuracy [4]. Such a setup has been studied in Refs. [19, 20] and is shown in Fig. 1(d). Taking into account a finite coupling efficiency for the emitted photon into the desired mode, and finite transmission and single-photon detection efficiencies, the generated joint state of atom and light is of the form $\rho_N^{at}(\eta_c\eta_t\eta_d)$, with

$$\rho_N^{at}(\eta) = [\cos(\theta)|g\rangle|vac\rangle_{N-1} + \sqrt{\eta}\sin(\theta)|s\rangle|W_{N-1}\rangle] \otimes h.c. + (1 - \eta)|s\rangle|vac\rangle \otimes h.c., \quad (4)$$

where the coefficient $\cos(\theta)$ is determined by the strength of the initial excitation pulse and $h.c.$ denotes hermitian conjugate. For the $N - 1$ optical modes, the protocol proceeds exactly as in the previous sections, with measurements of the type in Fig. 1(b). For the atomic party we allow projective measurements along any arbitrary direction on the Bloch sphere, and, to start with, we take the detection efficiency to be unity. For the bipartite setting $N = 2$, in the case of qubits and perfect Pauli measurements, one can easily find the critical efficiency analytically using the necessary and sufficient condition for violation of CHSH given in Ref. [21]. We find that $\tilde{\eta} = 1/2$ with an optimal angle of $\theta \rightarrow 0$. This was also obtained in Ref. [22] for polarization entanglement. For $N > 2$, we have verified numerically that the bound is unchanged. Remarkably, it is possible to reach this bound exactly using the realistic, displacement-based measurements considered above, for any number of parties. As before, we analytically compute $\xi(\mathbf{s})$ for the state $\rho_N^{at}(\eta)$, then numerically maximise the left-hand side of (1) and

vary η to obtain the critical efficiency. We find that $\tilde{\eta} = 50\%$ independent of N . In the bipartite case, it is possible to improve this number further by allowing a third measurement setting $A_2^{(k)}$ for both parties. We have tested the I3322 inequality [23], which applies to this scenario, and we find $\tilde{\eta} = 43.7\%$. An additional measurement setting does not complicate the experiment. Thus, this is the most attractive setup to implement.

The second variant of our scheme aims to eliminate transmission and source-coupling losses by the use of local filtering. A similar idea was the basis for Ref. [24], where it was used to improve Bell inequality violation as the basis for quantum key distribution. Since the effect of loss is to decrease the single-photon component of the final state, we let each party apply the probabilistic single-photon amplifier, proposed by Ralph and Lund [25] as illustrated in Fig. 1(e). We can model the filter, acting on each mode of the state received by the parties, as a quantum channel. A Kraus representation of this channel is given in the Appendix. Taking the limit $t \rightarrow 0$ and renormalising, we find that a successful application of the filter by every party takes $\rho_N(\eta_c \eta_t \eta_d) \rightarrow \rho_N(\eta'_c \eta_d)$. That is, successful filtering compensates transmission and coupling loss between the source and the parties but introduces an additional loss, corresponding to the coupling inside the filter. Thus, as long as $\eta'_c < \eta_c \eta_t$, it is beneficial to apply the filters. For example, if the same single-photon generation is employed for the initial shared source and the filter (e.g. parametric down-conversion as in Fig. 1(c)), $\eta'_c = \eta_c$ and it will be beneficial to filter as soon as there is significant transmission loss. However, successful preselection requires successful filtering by all parties simultaneously, and hence for a given success probability p_f , the rate of the experiment will decrease as p_f^N . Longer data collection times become necessary as the number of parties increases, and detector dark counts may become problematic.

As an interesting and experimentally attractive last setup, we consider the combination of the two extensions – atom-photon entanglement and local filtering. The filtering is applied only to the photonic modes. For the bipartite setting, we find that by choosing an appropriate small value of θ , for efficiencies close to the threshold $\tilde{\eta} = 43.7\%$, it is indeed possible to replace $\rho_2^{at}(\eta_c \eta_t \eta_d) \rightarrow \rho_2^{at}(\eta'_c \eta_d)$. This is a very nice result. It says that, by applying local filtering, it is possible to break I3322 for *any* value of the atom-photon coupling efficiency, as long as the combined detection and single-photon production efficiency employed by the photonic party fulfills $\eta'_c \eta_d > 43.7\%$. For a bipartite test, a small value of the preselection probability p_f is not critical, and a probabilistic single-photon source can be used. Efficient probabilistic single-photon generation is likely to be much easier to achieve in experiment than a high collection efficiency for atomic emission. For trapped ion or atoms, a collection efficiency of $\eta_c = 50\%$ is an opti-

mistic value at current (a fraction of spontaneous emission into the cavity mode of 51% has been achieved [26], and a recent study predicts $\eta_c > 30\%$ for coupling into single-mode fibre [27]). On the other hand, single-photon sources based on parametric down conversion achieving $\eta'_c \sim 85\%$ have been demonstrated [17]. It thus becomes possible to perform the Bell test with detectors of very moderate efficiencies $\eta_d = 0.437/\eta'_c \sim 51\%$. In a recent experiment, readout from a single trapped atom with efficiency exceeding 98% was demonstrated [28]. The detection time was ~ 800 ns implying that a separation of a few hundred metres is sufficient to close the locality loophole. We remark that detection efficiencies up to 99.99% have been demonstrated for trapped ions [29].

Conclusion.—In summary, we have presented a scheme for loophole-free violation of local realism based on single-photon W-states and simple measurements consisting of displacements followed by single-photon detection which does not need to resolve the photon number. We find favourable thresholds for the combined coupling, transmission, and detection losses with an asymptotic limit of $\tilde{\eta} \sim 68\%$ for large numbers of parties and $\tilde{\eta}$ in the 70s% range for moderate $N \lesssim 10$. We have extended the scheme to atom-photon entanglement and have shown that a threshold of $\tilde{\eta} = 47.3\%$ is achievable for $N = 2$, while for an arbitrary number of parties $\tilde{\eta} = 50\%$. Local filtering can compensate losses at the source, and in particular we have seen that the use of local filters based on parametric down conversion allows violations for an arbitrary small atom-photon coupling. The present scheme is implementable by currently available techniques, and thus we believe this work paves a feasible way towards experimental violation of local realism closing, simultaneously, both the detection and locality loopholes.

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Appendix.—Augmenting the circuit in Fig. 1(e) with additional vacuum modes and modeling losses by beam splitters, we can derive Kraus operators [30]. We find that in the 0-1-photon subspace, up to normalisation, the channel is described by the four operators

$$\begin{aligned} K_1 &= \sqrt{\frac{\eta_d(1-\eta'_c\eta_d)}{2}}|0\rangle\langle 1|, \\ K_2 &= -\sqrt{\frac{(1-t)\eta'_c\eta_d^2}{2}}|0\rangle\langle 1|, \\ K_3 &= -\sqrt{\frac{(1-t)\eta'_c\eta_d(1-\eta_d)}{2}}|0\rangle\langle 1|, \\ K_4 &= -\sqrt{\frac{(1-t)\eta'_c\eta_d}{2}}|0\rangle\langle 0| + \sqrt{\frac{t\eta'_c\eta_d^2}{2}}|1\rangle\langle 1|. \end{aligned} \quad (5)$$

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